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SOIL PHYSICS

Incorporation of Water Content in the Weibull Model for Soil Aggregate Strength

L. J. Munkholm*
Dep. of Agroecology and Environment
Faculty of Agricultural Sciences
Aarhus Univ.
Research Centre Foulum
P.O. Box 50
DK-8830 Tjele
Denmark

E. Perfect
Dep. of Earth and Planetary Sciences
Univ. of Tennessee
1412 Circle Dr.
Knoxville, TN 37922-1410

J. Grove
Dep. of Agronomy
Univ. of Kentucky
Agricultural Science Bldg. North
Lexington, KY 40546-0091

Tillage impacts many components and functions of the soil ecosystem. Thus, the prediction of soil structures produced by tillage may be regarded as a major objective in soil science. Brittle fracture is the desired mode of failure in most tillage operations. Mechanistic or phenomenological models based on the probabilistic Weibull "weakest link" theory are commonly applied to model brittle fracture of air-dry aggregates. The overall objective of this study was to develop a Weibull model to describe the strength of different-sized soil aggregates across a wide range of water contents. Rupture energy data were obtained for aggregates sampled in three field experiments. These included two soil compaction experiments (Bygholm I and II, sandy loam) and a long-term tillage and fertilization experiment (Maury, silt loam). Aggregates were subjected to a crushing test after having been adjusted to matric potentials ranging from −10 kPa to −163 MPa (air dry). Water content strongly affected the characteristic rupture energy (Weibull α parameter), and this relationship was successfully modeled with a power law function. In contrast, water content had little or no effect on the spread of aggregate strengths (Weibull β parameter). Based on these results, we proposed a three-parameter Weibull brittle fracture model for the tested sandy loam and silt loam soils that takes account of the effect of water content for a single aggregate size fraction. This model, in which only α depends on water content, explained on average 89% of the total variation in rupture energy. Further research is needed to fully investigate the influence of water content on the rupture energy of different-sized aggregates.

Abbreviations: AIC, Akaike's information criterion; cdf, cumulative probability density function; MP-N1, moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4, moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1, no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4, no-tillage and 336 kg N ha^{-1} yr^{-1} treatment; PAC, compacted treatment; REF, reference treatment.

A major purpose of tillage is to produce a favorable seedbed for plant growth, i.e., a collection of medium-sized aggregates (Braunack and Dexter, 1989). As a result, soil structures produced by tillage are of considerable interest from a crop production point of view. Furthermore, tillage has a pronounced effect on many soil ecosystem processes and functions. Despite the fundamental importance of tillage in soil science, there has been very little integrative work on the prediction of soil structures produced by tillage. Dexter (1979) stated more than 25 yr ago that "one of the principal aims of tillage research is to be able to predict what will be the effects on the soil of using a given tillage implement in given soil conditions." This still remains as a valid and timely objective in soil science.

Numerous studies have investigated individual aspects of tillage effects on soil structure and function. Little has been done, however, to synthesize existing knowledge into a model for predicting the soil structures produced by tillage. This may be due in part to the inherent complexity of the topic. Soil structures produced by tillage depend on many interconnected factors such as mode of failure, soil type, initial macrostructure, soil strength, and, not least, water content. Dexter (1979) presented an empirical model for predicting the fragment size distribution produced by tillage from a priori information on water content, implement type, etc. This model was developed specifically for an Australian sandy loam, however, and is not easily applied to other soils. Recently, Dexter and Birkas (2004) and Keller et al. (2006) have made an attempt to predict the amount of clods and smaller aggregates produced by tillage from Dexter's *S* index of soil physical quality. They have obtained good correlations but only made predictions for the optimum water content for tillage.

Brittle fracture (i.e., failure under expansion) is the desired mode of failure in most tillage operations, as it results in an overall loosening of the soil. Given that brittle fracture is the dominant mode of failure in tillage, the fragmentation produced by tillage may be predicted using brittle fracture theory. Mechanistic or phenomenological models based on the probabilistic Weibull "weakest link" theory (Weibull, 1952) have been applied to model brittle fracture of soil aggregates (Braunack et al., 1979; Perfect and Kay, 1995). In this model, the probability density function of aggregate strength is expressed as dependent on a deterministic size effect and a stochastic residual term.

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*Corresponding author(lars.munkholm@agrsci.dk).

The influence of soil type and management on the Weibull parameters has been extensively investigated in studies using air- or oven-dry aggregates (e.g., Hadas, 1987; Perfect and Blevins, 1997). To our knowledge, no previous studies have looked into the effect of water regime on the Weibull parameters. The relationship between tensile strength and water regime has been investigated in a number of studies using disturbed or undisturbed core samples (e.g., Snyder and Miller, 1985; Mullins et al., 1992; Panayiotopoulos, 1996; Aluko and Koolen, 2001). The relationship between aggregate tensile strength and water regime, however, has only been explored in a few studies (Guérif, 1990; Causarano, 1993; Chan, 1995; Munkholm and Kay, 2002), and none of these used the Weibull model to characterize aggregate strength. To our knowledge, the effect of water regime on aggregate size effects has not been explored before.

In this study, the overall objective was to develop a model to describe the strength of different-sized aggregates across a wide range of water contents. Such a model is needed to be able to predict soil structures produced by tillage. Our aim was to develop and evaluate a Weibull model for brittle fracture of aggregates that takes into account variations in soil water content. We addressed the effect of water content on the Weibull model parameters for individual size fractions and also investigated the effect of water content on the aggregate size effect.

THEORETICAL BACKGROUND

Rupture energy, E^*, has been recommended over tensile strength for the statistical characterization of aggregate strength because it is a measure of the strain energy applied until tensile failure even though some of the energy was used for plastic deformation rather than crack opening (Perfect and Kay, 1995). Additionally, E^* can be directly related to the energy input during tillage or, for instance, a drop shatter test (Munkholm et al., 2002). The rupture energy is obtained by integrating the stress–strain curve until the point of failure (Vomocil and Chancellor, 1967):

$$E^* = \int_0^z F(s)\,\mathrm{d}s \qquad [1]$$

where $F(s)$ is the compressive force at a specific strain and z is the strain at rupture. The specific rupture energy, E, may be defined on a gravimetric basis whereby no measure of the aggregate diameter is needed, i.e.,

$$E = E^*/m \qquad [2]$$

where m is the oven-dry mass of the individual aggregate.

According to the most widespread Weibull model, the probability of failure for a population of identically sized soil aggregates is given by (Perfect and Kay, 1995)

$$P\left(E \le E_i\right) = 1 - \exp\left[-\left(E/\alpha\right)^{\beta}\right] \qquad [3]$$

where $P(E \le E_i)$ is the cumulative probability density function, cdf, E_i are specified specific rupture energies, α is the characteristic rupture energy of the population, corresponding to the 63rd percentile of the cdf for E, and β characterizes the spread of rupture energies around α. The model presented in Eq. [3] is a two-parameter Weibull model as recommended by Munkholm and Perfect (2005). For a population of different-sized aggregates, the probability of failure can be calculated by (Perfect and Kay, 1995)

$$P\left(E \le E_i\right) = 1 - \exp\left[-\left(x/x_j\right)^{D}\left(E/\alpha\right)^{\beta}\right] \qquad [4]$$

where x is the length of the aggregate, x_j is the average length of the particle "building blocks," and D is a size scaling factor, which Perfect and Kay (1995) interpreted as the mass fractal dimension. The probability of failure in Eq. [1] and [2] is usually approximated by (Perfect and Kay, 1995)

$$P\left(E \le E_i\right) = l/\left(n+1\right) \qquad [5]$$

where l is the rank (in ascending order) of each measurement of E within a size fraction and n is the total number of samples for that fraction.

Numerous researchers have shown that aggregate tensile strength (Y) increases with decreasing water content. This can be ascribed to an increase in the cohesive forces of capillary-bound water by decreased pore water pressure (Bishop, 1961), as well as to increased effectiveness of cementing materials (Caron and Kay., 1992). Mullins and Panayiotopoulos (1984) proposed the following relationship between Y and matric potential, ψ:

$$Y = c - \chi\psi \qquad [6]$$

where c is cohesion and χ is a factor related to the degree of saturation. In the absence of externally applied stress, the $\chi\psi$ term describes the effective stress experienced by the soil. Mullins et al. (1992) showed a linear relationship between Y and ψ according to Eq. [6] for matric potentials in the range 0 to −100 kPa in a study using remolded core samples from a hardsetting Australian soil. For drier soil, the effective stress theory overpredicted Y. The lack of linearity below −100 kPa was explained by differences in the geometry of the pores involved in the tensile failure process when the soil dries, i.e., from spherical to more linear-shaped pores with decreasing water content. The creation of internal shrinkage cracks at low water contents was also proposed as a possible mechanism. A nonlinear relationship between soil strength and effective stress was also found by Aluko and Koolen (2000).

Snyder and Miller (1985) refined the effective stress approach to be able to describe tensile strength of unsaturated soils across a large range of water contents (0 to −1500 kPa). The refined approach worked fine for remolded soil cores but not cores with intact soil structure. They explained the latter by a difference in the part of the pore space contributing to tensile strength. For the cores with intact structure, only a small part of the pore space will be involved because tensile failure is expected to occur at interaggregate points. In addition, their refined effective stress approach did not take into account cementation, which is expected to be of importance under dry conditions and especially for undisturbed soils (Snyder and Miller, 1985).

Empirical correlations between aggregate Y and volumetric water content, θ, or ψ have been found in a number of studies (Guérif, 1988; Causarano, 1993; Chan, 1995; Munkholm and Kay, 2002). The latter researchers observed that, for -166 MPa $< \psi < -10$ kPa, the relationship between Y and ψ could be expressed by a power function:

$$Y = -q\psi^{n} \qquad [7]$$

where q and n are regression coefficients. Munkholm and Kay (2002) found that the relationship between E and ψ could also be fitted by a power function for one of the two soils included in their study.

To our knowledge, incorporation of the effect of water content into a Weibull brittle fracture model has not been attempted previously. To facilitate this process, we decided to work with the gravimetric water content, w, instead of θ or ψ. This is because both θ and ψ are difficult to measure on individual soil aggregates, and as a result are generally unavailable. In contrast, w is routinely measured in soil aggregate strength studies, and as a result a modified Weibull model based on w should gain widespread acceptance and be applicable to data sets already collected.

MATERIALS AND METHODS

Soils and Measurements

Aggregate strength data were analyzed for two different soils: Bygholm sandy loam (an Oxyaquic Agriudoll) and Maury silt loam (a Typic Paleudalf). The Bygholm samples were collected from two traffic compaction experiments (I and II) located on the organically managed Rugballegård Experimental Station, Denmark (Munkholm and Kay, 2002; Munkholm and Perfect, 2005). In both experiments, a compaction treatment (PAC) was compared with a reference treatment (REF). The samples were taken from the 7- to 12-cm and the 0- to 5-cm depths in Exp. I and II, respectively. Additional information regarding the field trials can be found in Munkholm and Kay (2002) (Exp. I) and Munkholm and Perfect (2005) (Exp. II). The bulked samples were air dried and separated into different aggregate size classes: 1 to 2, 2 to 4, 4 to 8, and 8 to 16 mm (Exp. I), and 2 to 4, 4 to 8, 8 to 16, and 16 to 32 mm (Exp. II) as described in Munkholm and Kay (2002). In this study, data from the two largest fractions for each experiment were used for evaluation of water content effect on aggregate strength for single size fractions. Data from all size fractions were used to investigate the effect of water content on the size scaling factor, D, in Eq. [4].

The Maury samples were collected from a long-term tillage experiment with maize (*Zea mays* L.) at the University of Kentucky Experiment Station research farm, near Lexington, KY. The experiment was established in 1970 as a split block design with four replicates. The blocks were split laterally for randomized N fertilizer treatments (N1: 0 kg N ha^{-1}; N2: 84 kg N ha^{-1}; N3: 168 kg N ha^{-1}; and N4: 336 kg N ha^{-1}) and longitudinally for randomized tillage treatments (moldboard plowing, MP, and no-tillage, NT). Plowing was done to a depth of 20 to 25 cm, followed by disking to a depth of 8 to 10 cm each spring. Further details on the experimental design and its influence on soil properties can be found in Perfect and Blevins (1997) and Perfect and Caron (2002). In this study, samples were taken in the MP and NT plots receiving either N1 or N4. In each plot, four undisturbed soil samples were collected in untrafficked interrows from the 0- to 10-cm depth using a cylindrical core sampler (diameter: 5.2 cm). The samples were placed in plastic bags in the field and kept cool until initial fragmentation. All the samples were dropped from 1.5-m height onto an aluminum tray. The shattered soil was air dried before size separation. A subsample was taken from each core for water content determination before dropping. Aggregates in the size range 4- to 8-mm aggregates were obtained by sieving (Munkholm and Kay, 2002).

Aggregates from each soil, treatment, and size fraction were adjusted to −10, −30, −100, and −350 kPa matric potential as described by Munkholm and Kay (2002). The aggregates were slowly wetted to −1.5 kPa on tension tables to minimize slaking and the generation of microcracks. Thereafter, the aggregates were adjusted to the required matric potential. A number of aggregates were taken out randomly and used in the crushing test. The remaining aggregates were subsequently dried to −30, −100, and −350 kPa using pressure plates. For the Bygholm II soil, the −350-kPa matric potential was not used. For the Maury soil, aggregates were also adjusted to −4.5 MPa matric potential (assuming no salts in the soil solution) using the controlled vapor pressure method (Nimmo and Winfield, 2002). Aggregates, initially adjusted to −30 kPa according to Munkholm and Kay (2002), were transferred to a closed chamber that contained an open vessel with a 1 *M* NaCl solution. The aggregates were placed in the chamber for 3.5 mo at a constant temperature of 20°C. A few of the samples were not satisfactorily equilibrated after 3.5 mo due to problems with the formation of condensed water in the chamber; these samples were excluded from all subsequent analyses. Rupture energy was also determined on air-dry aggregates. The matric potential in air-dry soil was calculated to be −163 MPa, assuming the aggregates were in equilibrium with the air in the laboratory, which had a constant temperature of 20°C and a relative humidity of approximately 30% (Kutilek and Nielsen, 1994, Eq. [4.35]).

Following equilibration at a given matric potential, selected aggregates from each size fraction were crushed individually between two flat parallel plates at constant rate of displacement of 2 mm min^{-1} using the indirect tension test. Rupture energy, E, was determined according to Munkholm and Kay (2002). The weights of the individual aggregates were recorded and adjusted to oven-dry weight to give m. The weights of the smallest aggregates of the Bygholm experiments were estimated as described in Munkholm and Kay (2002). The E^* was calculated from E and m using Eq. [2]. Generally, a linear stress–strain relationship until failure was found at all water contents and the aggregates generally broke into two halves along the vertical axis. This indicates that tensile failure was the dominant mode of failure. The smallest size fractions at the highest water content for the Bygholm soils were excluded, as they showed a tendency to compress rather than fail in tension. For the Bygholm soils, 15 aggregates were crushed for each combination of soil, matric potential, and size class with the following exceptions: the 1- to 2-mm aggregates were not used at −10 and −30 kPa, and the 2- to 4-mm aggregates were not used at −10 kPa. For the Maury soil, 30 aggregates were used for each combination of treatment (tillage and fertilizer) and matric potential. In all, approximately 5400 aggregates were subjected to the crushing test.

Data Analysis

The $P(E \le E_i)$ was calculated from the measured rupture energies using Eq. [5]. The ranking was performed for data from individual plots for the Maury soil, i.e., 30 samples per plot. For the Bygholm soils, the data for each combination of trial, treatment, and size were pooled to provide an adequate number of samples for parameter estimation (i.e., 45 samples after pooling). The individual α and β Weibull parameters for the different combinations of soil, water content, size fraction, and treatment were estimated by fitting Eq. [3] to the $P(E \le E_i)$ and E data by nonlinear regression analysis (PROC NLIN, SAS Institute, 1999), as described by Munkholm and Perfect (2005). Relationships between water content and the Weibull parameters α and β were explored using either linear or nonlinear regression. The parameters in the proposed Weibull models incorporating water content were estimated using nonlinear regression. All of the nonlinear fits converged according to the software default criterion (SAS Institute, 1999). The balance between goodness-of-fit and parsimony for the different models was evaluated using Akaike's information criterion (AIC) (SAS Institute, 1999):

$$\mathrm{AIC} = n \ln\left(\mathrm{RSS}/n\right) + 2p \qquad [8]$$

where n is the number of observations, RSS is the residual sums of squares, and p is the number of model parameters. The smaller (more negative) the AIC value, the better the model. The parameter esti-

Table 1. Average gravimetric water content, w, and relative water content, w/w_0, at the different applied matric potentials, Ψ.

Soil/ experiment	Treatment†	Size	Matric potential, Ψ: −166 MPa w	w/w_0	−4.5 MPa w	w/w_0	−350 kPa w	w/w_0	−100 kPa w	w/w_0	−30 kPa w	w/w_0	−10 kPa w	w/w_0
		mm	g kg^{-1}		g kg^{-1}		g kg^{-1}		g kg^{-1}		g kg^{-1}		g kg^{-1}	
Bygholm I	PAC	4–8	15	1	nd‡	nd	84	5.6	131	8.8	163	11.1	184	12.4
	REF	4–8	14	1	nd	nd	82	6.0	125	9.1	161	11.7	187	13.6
	PAC	8–16	15	1	nd	nd	85	5.6	127	8.4	179	11.8	184	12.2
	REF	8–16	14	1	nd	nd	77	5.5	121	8.5	177	12.5	187	13.2
Bygholm II	PAC	8–16	13	1	nd	nd	nd	nd	126	9.5	169	12.8	197	14.9
	REF	8–16	13	1	nd	nd	nd	nd	119	9.5	167	13.3	193	15.4
	PAC	16–32	14	1	nd	nd	nd	nd	127	9.2	165	11.9	188	13.6
	REF	16–32	14	1	nd	nd	nd	nd	123	8.9	167	12.1	190	13.7
Maury§	MP-N1	4–8	24	1	85	3.5	134	5.6	178	7.4	210	8.8	228	9.5
	MP-N4	4–8	26	1	76	3.0	133	5.2	180	7.0	218	8.5	245	9.5
	NT-N1	4–8	24	1	89	3.8	139	5.7	216	8.9	254	10.5	283	11.7
	NT-N4	4–8	25	1	100	4.0	141	5.7	219	8.9	272	11.0	305	12.4

† PAC, compacted treatment; REF, reference treatment; MP-N1, moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4, moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1, no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4, no-tillage and 336 kg N ha^{-1} yr^{-1} treatment.

‡ Not determined.

§ Averaged across blocks.

mates and the AIC values obtained for each proposed model and data set were subsequently used for statistical analysis of effect of model using ANOVA and comparison of means techniques in PROC GLM (SAS Institute, 1999).

RESULTS AND DISCUSSION

Water Content Effect on Weibull Parameters

An empirical approach was followed to investigate the influence of water content on the Weibull parameters α and β. In this study, the air-dry water content, w_0, was set as a reference point. This was done because aggregate strength reaches its maximum for the air-dry condition, and because past investigations estimating Weibull parameters have largely used air-dry soil. The relative water content was then calculated as w/w_0. Data for w and w/w_0 and the different adjusted ψ values are presented in Table 1. To simplify, averages across blocks for the Maury treatments are shown. The exact values of w/w_0 per plot were used when estimating the effect of water content on the Weibull parameters.

The relationships between w/w_0 and the characteristic rupture energy, α, are shown in Fig. 1. For clarity, only data for the two largest size fractions from the Bygholm studies and from Block 2 of the Maury experiment are shown. The relationship between α and w/w_0 could be modeled by a power law function:

$$\alpha = \alpha_0 \left(w/w_0\right)^{\gamma} \qquad [9]$$

where α_0 is the characteristic rupture energy for the air-dry condition and γ is a water content scaling factor. Excellent fits of Eq. [9] were obtained for all of the soils and treatments with coefficients of determination, R^2, between observed and predicted values ranging from 0.93 to 0.98, 0.99 to 1.00, and 0.48 to 0.99 for Bygholm I, Bygholm II, and Maury soils, respectively (Table 2). The generally lower R^2 values for the Maury soil are partly due to unpredicted low α values at w/w_0 between 3 and 4 (Fig. 1c), which corresponds to the measurements at ψ = −4.5 MPa (Table 2). In fact, there appears to be a local minimum at w/w_0 = 3 to 4 (i.e., measurements at ψ =

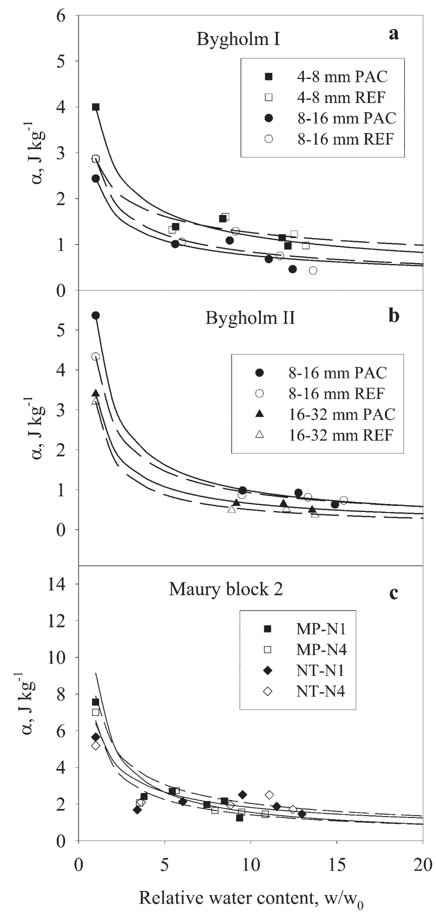

Fig. 1. Relationship between characteristic rupture energy, α, and the relative water content (w/w_0) for (a) Bygholm I soil, (b) Bygholm II soil, and (c) Maury soil Block 2 aggregates. Lines correspond to the fitted power law relationships summarized in Table 2. PAC = compacted treatment; REF = reference treatment; MP-N1 = moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4 = moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1 = no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4 = no-tillage and 336 kg N ha^{-1} yr^{-1} treatment.

Table 2. Regression coefficients and R^2 values for the model $\alpha = \alpha_0(w/w_0)^\gamma$, where α_0 is the characteristic rupture energy for the air-dry condition, w/w_0 is relative water content, and γ is a water content scaling factor (Eq. [9]), fitted using nonlinear regression.

Soil/ experiment	Treatment†	Block	n‡	Size	α_0	γ	R^2
				mm	J kg^{-1}		
Bygholm I	PAC	all	45	4–8	3.98 (0.22)§	−0.52 (0.05)	0.98***
	REF	all	45	4–8	2.85 (0.22)	−0.36 (0.05)	0.93***
	PAC	all	45	8–16	2.45 (0.20)	−0.51 (0.07)	0.95**
	REF	all	45	8–16	2.87 (0.28)	−0.54 (0.09)	0.93***
Bygholm II	PAC	all	45	8–16	5.36 (0.10)	−0.74 (0.03)	1.00***
	REF	all	45	8–16	4.33 (0.08)	−0.67 (0.02)	1.00***
	PAC	all	45	16–32	3.40 (0.06)	−0.72 (0.03)	1.00***
	REF	all	45	16–32	3.20 (0.07)	−0.81 (0.04)	1.00***
Maury	MP-N1	1	30	4–8	14.27 (0.68)	−1.08 (0.09)	0.98***
	MP-N1	2	30	4–8	7.50 (0.44)	−0.69 (0.06)	0.97***
	MP-N1	3	30	4–8	9.00 (0.70)	−0.75 (0.09)	0.95***
	MP-N1	4	30	4–8	5.85 (0.40)	−0.47 (0.05)	0.95***
	MP-N4	1	30	4–8	8.45 (0.30)	−0.83 (0.05)	0.99***
	MP-N4	2	30	4–8	6.89 (0.52)	−0.69 (0.08)	0.95***
	MP-N4	3	30	4–8	3.33 (0.58)	−0.25 (0.12)	0.48**
	MP-N4	4	30	4–8	7.46 (0.85)	−0.78 (0.15)	0.89**
	NT-N1	1	30	4–8	9.14 (0.57)	−0.60 (0.06)	0.97***
	NT-N1	2	30	4–8	5.35 (0.73)	−0.51 (0.11)	0.82**
	NT-N1	3	30	4–8	4.71 (0.67)	−0.42 (0.10)	0.77**
	NT-N1	4	30	4–8	6.44 (0.91)	−0.59 (0.13)	0.82**
	NT-N4	1	30	4–8	11.57 (1.00)	−0.72 (0.09)	0.94***
	NT-N4	2	30	4–8	4.97 (0.60)	−0.42 (0.10)	0.86**
	NT-N4	3	30	4–8	7.55 (1.02)	−0.58 (0.12)	0.85**
	NT-N4	4	30	4–8	7.48 (1.08)	−0.58 (0.13)	0.82**

** Significant at the $P < 0.01$ level.

*** Significant at the $P < 0.001$ level.

† PAC, compacted treatment; REF, reference treatment; MP-N1, moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4, moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1, no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4, no-tillage and 336 kg N ha^{-1} yr^{-1} treatment.

‡ Number of observations at each water content.

§ Approximate standard errors in parentheses.

−4.5 MPa) even though these values were not significantly (P > 0.05) lower than those at w/w_0 = 5 to 6 (i.e., measurements at ψ = −350 kPa). The unpredicted low α values at w/w_0 = 3 to 4 may be due to a drop in the cohesive forces of capillary bound water at this low degree of saturation, i.e., only pores less than ~0.06 µm in diameter will be water filled at ψ = −4.5 MPa.

Table 2 indicates that the α_0 parameter was affected by soil, aggregate size, and experimental treatment as expected from other studies (e.g., Braunack et al., 1979; Perfect and Kay, 1994). The water content scaling factor, γ, varied markedly between soils, with the lowest values for the Bygholm I soil (0.36–0.54) and the highest values for the Bygholm II soil (0.67–0.81). The Maury soil results indicate a stronger decrease in rupture energy on wetting for the MP treatments than NT although the difference was not significant (P = 0.14; average γ = −0.69 and −0.55 for MP and NT, respectively). The MP treatments contained the lowest organic matter contents (Perfect and Caron, 2002). This tentative effect of organic matter on the increase in aggregate strength on drying is in accordance with previous findings by Chan (1995), Munkholm and Kay (2002), and Munkholm et al. (2002). In general, a soil with little increase in strength on drying is desirable, i.e., with stable aggregates in the wet condition and friable aggregates in the dry condition. A strong increase in strength on drying may restrict the timing and quality of tillage and limit root growth under dry conditions. This is a characteristic of hardsetting soils, which are very difficult to manage (Mullins et al., 1987). Besides organic matter, texture and clay mineralogy are also known to affect the potential of hardsetting behavior. Interestingly, we found marked differences between the different Bygholm soils used in this study even though they were similar regarding organic matter content, texture, and clay mineralogy (data not shown). It is possible that the time of sampling may have played a role, since the severity and duration of wetting events before sampling has been shown to significantly affect aggregate strength (Kay et al., 1994).

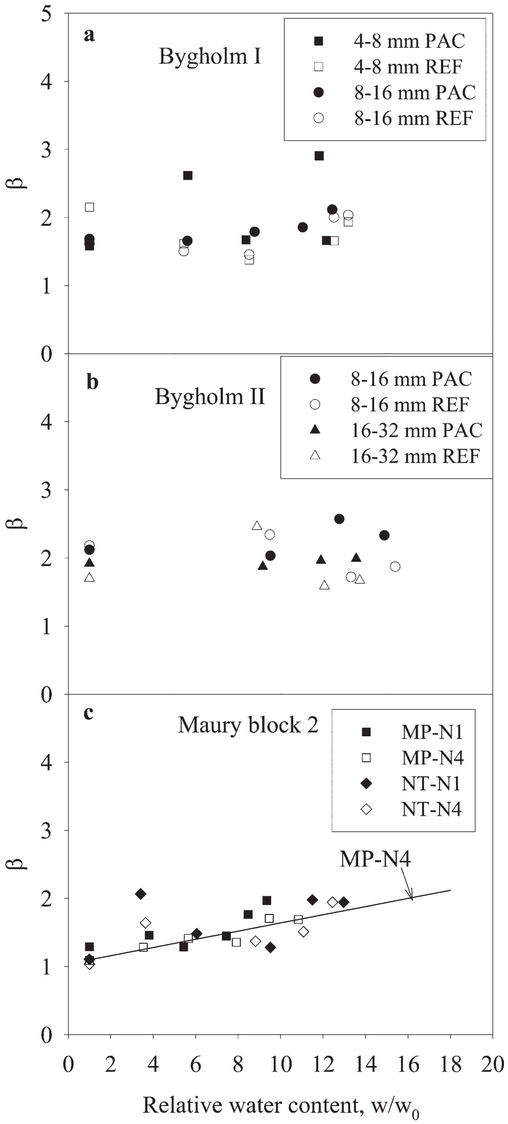

Fig. 2. Relationship between the spread of rupture energy Weibull parameter, β, and the relative water content (w/w_0) for (a) Bygholm I soil, (b) Bygholm II soil, and (c) Maury soil Block 2 aggregates. Line for MP-N4 corresponds to the significant linear relationship summarized in Table 3. PAC = compacted treatment; REF = reference treatment; MP-N1 = moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4 = moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1 = no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4 = no-tillage and 336 kg N ha^{-1} yr^{-1} treatment.

The dependency of the β parameter on w/w_0 is illustrated in Fig. 2. For the Maury soil, the relationship could, in some cases, be described by a simple linear model:

$$\beta = \beta_0 + k(w/w_0) \quad [10]$$

where k is a regression parameter describing the relationship between β and w/w_0. For most of the Maury data set, β increased slightly ($k \le 0.14$) with w/w_0—although the relationship was only significant ($P < 0.05$) for 3 out of 16 data sets (Table 3). For the Bygholm soils, there was also a tendency—although not significant—for a slight increase in β with relative water content in most cases. The parameter $1/\beta$ is a measure of the spread of strengths and has been proposed as a friability index (Utomo and Dexter, 1981; Perfect and Kay, 1994). Estimates of $1/\beta$ based on aggregate strength measurements have shown a tendency to display maximum values between −30 and −100 kPa (e.g., Utomo and Dexter, 1981; Munkholm and Kay, 2002). The latter researchers also estimated $1/\beta$ from E measurements using the Bygholm I soils and found that $1/\beta$ increased with increased water content. Therefore, the trend in this study of a slight increase in β with increasing relative water content is surprising. Differences in the methods used to estimate β may explain, to some extent, this divergence. In the present study, β was estimated from individual size fraction data, whereas β was estimated across size fractions in the study of Munkholm and Kay (2002).

Our results show that β was relatively more sensitive to soil and size than to water content. The lowest β values (i.e., greatest spread of rupture energies) were generally found for the Maury soil and highest for the Bygholm II soil. For the Bygholm soils, the highest values of β were generally found for the largest aggregate fractions. The insensitivity of β to water content indicates that, for any given size fraction, the wetting and drying treatments used affected both weak and strong aggregates alike. This observation further suggests that slaking was largely eliminated. If rapid wetting had occurred, it would probably have preferentially disrupted the weakest aggregates, thereby decreasing the range of strengths, increasing β, and introducing a trend with equilibration potential or water content.

Inclusion of Water Content into the Weibull Model

For the Maury soil, both α and β varied significantly with water content, although α much more strongly than β. From a modeling perspective, the question is whether it is worthwhile to include dependence of water content for both α and β. Therefore, we compared a four-parameter Weibull model (Eq. [11]), where both α and β vary with water content, with a three-parameter model that takes into account only α's dependence on water content (Eq. [12]):

$$P(E \le E_i) = 1 - \exp\left(-\left\{E\Big/\left[\alpha_0 (w/w_0)^{\gamma}\right]\right\}^{\beta_0 + k(w/w_0)}\right) \quad [11]$$

$$P(E \le E_i) = 1 - \exp\left(-\left\{E\Big/\left[\alpha_0 (w/w_0)^{\gamma}\right]\right\}^{\beta}\right) \quad [12]$$

These two models were only compared for the Maury soil, since there was no significant effect of water content on the β parameter for the two Bygholm soils. Both models explained a large degree of the total variation in $P(E \le E_i)$ (Tables 4 and 5). The R^2 values varied from 0.75 to 0.92 and 0.79 to 0.92 for the three- and four-parameter models, respectively. On average, inclusion of a β vs. water content relationship (Eq. [12]) increased the R^2 value by only ~1%, i.e., from 87 to 88%. The mean AIC value decreased from −802 to −815, indicating that the four-parameter model was slightly better. Although, the difference in mean AIC values between the models was significant ($P = 0.01$), increasing the number of parameters from three to four had no significant effect ($P > 0.05$) on estimates of α_0 and γ. This is apparent from the approximate standard errors listed in Tables 4 and 5. As expected, the β_0 estimates for the four-parameter model were generally lower than the β estimates

Table 3. Regression coefficients and R^2 values for the model $\beta = \beta_0 + k(w/w_0)$, where β is the spread of aggregate strengths, k is the regression parameter describing the relationship between β and w/w_0, and w/w_0 is relative water content (Eq. [10]), fitted using linear regression.

Soil/experiment	Treatment†	Block	n‡	Size	β_0	k	R^2
				mm			
Bygholm I	PAC	all	45	4–8	1.85 (0.68)§	0.03 (0.08)	0.05 NS
	REF	all	45	4–8	1.94 (0.30)	−0.02 (0.03)	0.15 NS
	PAC	all	45	8–16	1.36 (0.22)	0.05 (0.02)	0.72 NS
	REF	all	45	8–16	1.40 (0.20)	0.04 (0.02)	0.52 NS
Bygholm II	PAC	all	45	8–16	2.04 (0.25)	0.02 (0.02)	0.34 NS
	REF	all	45	8–16	2.30 (0.27)	−0.03 (0.02)	0.25 NS
	PAC	all	45	16–32	1.89 (0.06)	0.00 (0.01)	0.25 NS
	REF	all	45	16–32	1.90 (0.52)	−0.01 (0.05)	0.01 NS
Maury	MP-N1	1	30	4–8	1,21 (0.16)	0.04 (0.03)	0.28 NS
	MP-N1	2	30	4–8	1.13 (0.17)	0.07 (0.03)	0.63 NS
	MP-N1	3	30	4–8	1.22 (0.18)	0.05 (0.03)	0.43 NS
	MP-N1	4	30	4–8	0.87 (0.33)	0.12 (0.04)	0.64 NS
	MP-N4	1	30	4–8	0.83 (0.11)	0.14 (0.02)	0.96**
	MP-N4	2	30	4–8	1.04 (0.08)	0.06 (0.01)	0.87**
	MP-N4	3	30	4–8	1.25 (0.19)	0.04 (0.03)	0.36 NS
	MP-N4	4	30	4–8	1.71 (0.09)	−0.01 (0.02)	0.17 NS
	NT-N1	1	30	4–8	1.30 (0.51)	0.07 (0.07)	0.25 NS
	NT-N1	2	30	4–8	1.36 (0.33)	0.04 (0.04)	0.20 NS
	NT-N1	3	30	4–8	1.30 (0.55)	0.07 (0.07)	0.20 NS
	NT-N1	4	30	4–8	1.39 (0.11)	0.03 (0.01)	0.59 NS
	NT-N4	1	30	4–8	2.22 (0.72)	0.01 (0.10)	0.00 NS
	NT-N4	2	30	4–8	1.15 (0.25)	0.05 (0.03)	0.48 NS
	NT-N4	3	30	4–8	1.15 (0.33)	0.06 (0.04)	0.36 NS
	NT-N4	4	30	4–8	1.32 (0.16)	0.07 (0.02)	0.78*

* Significant at the $P < 0.05$ level; NS = not significant.

** Significant at the $P < 0.01$ level.

† PAC, compacted treatment; REF, reference treatment; MP-N1, moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4, moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1, no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4, no-tillage and 336 kg N ha^{-1} yr^{-1} treatment.

‡ Number of observations at each water content.

§ Approximate standard errors in parentheses.

Table 4. Regression coefficients: characteristic rupture energy in the air-dry state (α_0), spread of aggregate strengths in the air-dry state (B_0), the water content scaling factor for α (γ), the regression parameter describing the relationship between β and w/w_0 (k), and R^2 and Akaike's information criterion (AIC) values for the four-parameter Weibull model (Eq. [11])—Maury soil.

Treatment†	Block	α_0	β_0	γ	k	R^2	AIC
		J kg^{-1}					
MP-N1	1	13.79 (0.76)‡	1.13 (0.12)	−1.01 (0.03)	0.02 (0.02)	0.87***	−821
MP-N1	2	7.32 (0.32)	1.23 (0.11)	−0.67 (0.02)	0.02 (0.02)	0.91***	−881
MP-N1	3	8.40 (0.37)	1.07 (0.09)	−0.68 (0.02)	0.05 (0.01)	0.91***	−882
MP-N1	4	5.91 (0.26)	1.04 (0.09)	−0.48 (0.02)	0.06 (0.01)	0.92***	−906
MP-N4	1	8.83 (0.47)	0.89 (0.09)	−0.85 (0.03)	0.10 (0.02)	0.92***	−754
MP-N4	2	6.44 (0.27)	0.90 (0.07)	−0.63 (0.02)	0.07 (0.01)	0.94***	−946
MP-N4	3	3.29 (0.17)	1.15 (0.10)	−0.22 (0.03)	0.01 (0.02)	0.87***	−826
MP-N4	4	6.68 (0.31)	1.07 (0.12)	−0.64 (0.03)	0.08 (0.02)	0.87***	−821
NT-N1	1	8.96 (0.37)	1.28 (0.12)	−0.59 (0.02)	0.05 (0.02)	0.91***	−732
NT-N1	2	4.68 (0.27)	0.90 (0.09)	−0.40 (0.03)	0.06 (0.01)	0.88***	−837
NT-N1	3	4.27 (0.30)	0.95 (0.11)	−0.33 (0.03)	0.05 (0.02)	0.83***	−770
NT-N1	4	5.43 (0.39)	0.93 (0.11)	−0.42 (0.04)	0.04 (0.02)	0.80***	−744
NT-N4	1	11.74 (0.38)	2.08 (0.19)	−0.63 (0.02)	−0.02 (0.02)	0.90***	−720
NT-N4	2	4.18 (0.21)	1.06 (0.09)	−0.32 (0.02)	0.04 (0.01)	0.91***	−889
NT-N4	3	6.53 (0.54)	0.65 (0.08)	−0.46 (0.04)	0.07 (0.01)	0.85***	−788
NT-N4	4	6.37 (0.53)	0.74 (0.11)	−0.42 (0.04)	0.08 (0.02)	0.79***	−731
Mean						0.88	−815

*** Significant at the $P < 0.001$ level.

† MP-N1, moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4, moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1, no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4, no-tillage and 336 kg N ha^{-1} yr−1 treatment.

‡ Approximate standard errors in parentheses.

Table 5. Regression coefficients: characteristic rupture energy in the air-dry state (α_0), spread of aggregate strengths (β), the water content scaling factor for α (γ), and R^2 and Akaike's information criterion (AIC) values for the three-parameter Weibull model (Eq. [12])—Maury soil.

Treatment†	Block	α_0	β	γ	R^2	AIC
		J kg^{-1}				
MP-N1	1	13.65 (0.70)‡	1.21 (0.05)	−1.01 (0.03)	0.87***	-822
MP-N1	2	7.19 (0.29)	1.36 (0.05)	−0.65 (0.02)	0.91***	−881
MP-N1	3	8.15 (0.32)	1.35 (0.05)	−0.66 (0.02)	0.90***	−873
MP-N1	4	5.54 (0.21)	1.42 (0.05)	−0.44 (0.02)	0.91***	−888
MP-N4	1	7.95 (0.39)	1.39 (0.06)	−0.78 (0.03)	0.90***	−728
MP-N4	2	6.26 (0.24)	1.29 (0.04)	−0.60 (0.02)	0.92***	−909
MP-N4	3	3.26 (0.16)	1.23 (0.05)	−0.21 (0.03)	0.87***	−827
MP-N4	4	6.52 (0.26)	1.43 (0.07)	−0.62 (0.03)	0.86***	−811
NT-N1	1	8.82 (0.33)	1.60 (0.07)	−0.57 (0.02)	0.90***	−726
NT-N1	2	4.30 (0.22)	1.31 (0.06)	−0.35 (0.03)	0.86***	−813
NT-N1	3	4.13 (0.26)	1.25 (0.06)	−0.31 (0.03)	0.82***	−763
NT-N1	4	5.25 (0.33)	1.19 (0.06)	−0.41 (0.03)	0.79***	−738
NT-N4	1	11.83 (0.37)	1.97 (0.09)	−0.64 (0.02)	0.90***	−722
NT-N4	2	3.80 (0.16)	1.41 (0.06)	−0.28 (0.02)	0.91***	−878
NT-N4	3	6.07 (0.44)	1.13 (0.06)	−0.41 (0.03)	0.81***	−750
NT-N4	4	5.93 (0.39)	1.29 (0.08)	−0.38 (0.03)	0.75***	−705
Mean					0.87	−802

*** Significant at the $P < 0.001$ level.

† MP-N1, moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment; MP-N4, moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment; NT-N1, no-tillage and 0 kg N ha^{-1} yr^{-1} treatment; NT-N4, no-tillage and 336 kg N ha−1 yr−1 treatment.

‡ Approximate standard errors in parentheses.

for the three-parameter model. All in all, inclusion of a β vs. water content relationship in the model slightly improved the degree of explanation but had no significant effect on the α_0 and γ parameters that provide information on the relationship between characteristic rupture energy and water content scaling, respectively. We therefore advocate use of the three-parameter model. For the Bygholm soils, this model described a substantial part of the total variation, i.e., R^2 was 0.82 to 0.94 and 0.94 to 0.97 for Bygholm I and Bygholm II, respectively (Table 6).

The extent to which the cdf curves collapse after normalization to air-dry condition provides a graphical measure of the goodness-of-fit for the proposed three-parameter model. That is, $P(E \le E_i)$ is depicted against E and $E/\alpha_0(w/w_0)^\gamma$, respectively. Note that $P(E \le E_i)$ vs. $E/\alpha_0(w/w_0)^\gamma$ should, according to Eq. [12], give a single curve for a given soil and aggregate-size combination, because β is constant and all the α values are scaled to the air-dry state. In general, the cdf curves collapsed very well for the Bygholm soil—some examples are shown in Fig. 3. This was the case across size fractions and treatments. Only for the 8- to 16-mm Bygholm I REF aggregates did the normalization procedure give an unsatisfactorily large deviation from the air-dry reference level (data not shown). This was also the Bygholm data set with the lowest R^2 value (R^2 = 0.82). For the Maury soil, where E was determined at more water contents, the cdf curves collapsed rather well after normalization in most cases, as illustrated with the results for Block 2 (Fig. 4). Remarkably, the ψ = −4.5 MPa (i.e., w/w_0 = 3–4) results display deviations from the air-dry results after normalization. This was the water potential or content where unpredicted low values of rupture energies were observed, as discussed above (Fig. 1c).

Water Content Influence on Aggregate Size Effect

Our results allowed calculation, using Eq. [4], of the size-scaling factor (D) for the Bygholm soils at the tested water contents and including data from all size fractions. We estimated low values of D at air-dry condition (i.e., between 1.1 and 1.3; Table 7). This is in accordance with the results of Perfect and Kay (1995) and Perfect et al. (1998). Our findings confirm that estimates of D from brittle fracture measurements are much lower than the estimates obtained from soil water retention measurements, where D is close to three (Perfect and Kay, 1995). As a result, estimates of D from brittle fracture measurements cannot be used as a proxy for the mass fractal dimension of a material. This discrepancy is perplexing, since Eq. [4] was derived using fractal geometry. It may be due in part to differences in the scaling of failure zones compared with that for pore volumes. Based on this interpretation, the observed values of D for soil at air-dry condition indicate that E scales predominantly to aggregate length ($D \sim 1$) rather than to cross-sectional area ($D \sim 2$) or volume ($D \sim 3$). Others have also found that aggregate strength depends more on aggregate length than volume (Braunack et al., 1979; Perfect and Kay, 1994).

Table 6. Regression coefficients: characteristic rupture energy in the air-dry state (α_0), spread of aggregate strengths (β, the water content scaling factor for α (γ), and R^2 values for the three-parameter Weibull model (Eq. [12])—Bygholm soils.

Soil/ experiment	Treatment†	Size	α_0	β	γ	R^2
		mm	J kg−1			
Bygholm I	PAC	4–8	3.91 (0.09)‡	1.78 (0.06)	−0.54 (0.01)	0.92***
	REF	4–8	2.95 (0.06)	1.62 (0.04)	−0.41 (0.01)	0.94***
	PAC	8–16	2.66 (0.10)	1.45 (0.06)	−0.59 (0.02)	0.86***
	REF	8–16	3.09 (0.14)	1.35 (0.06)	−0.63 (0.02)	0.82***
Bygholm II	PAC	8–16	5.40 (0.10)	2.08 (0.06)	−0.74 (0.01)	0.94***
	REF	8–16	4.35 (0.06)	1.99 (0.04)	−0.67 (0.01)	0.97***
	PAC	16–32	3.40 (0.05)	1.89 (0.04)	−0.72 (0.01)	0.97***
	REF	16–32	3.15 (0.07)	1.73 (0.05)	−0.80 (0.01)	0.94***

*** Significant at the $P < 0.001$ level.

† PAC, compacted treatment; REF, reference treatment.

‡ Approximate standard errors in parentheses.

The brittle fracture process itself may explain this. According to the tensile failure theory developed by Griffith (1920), tensile failure is caused by the propagation of cracks in the stressed

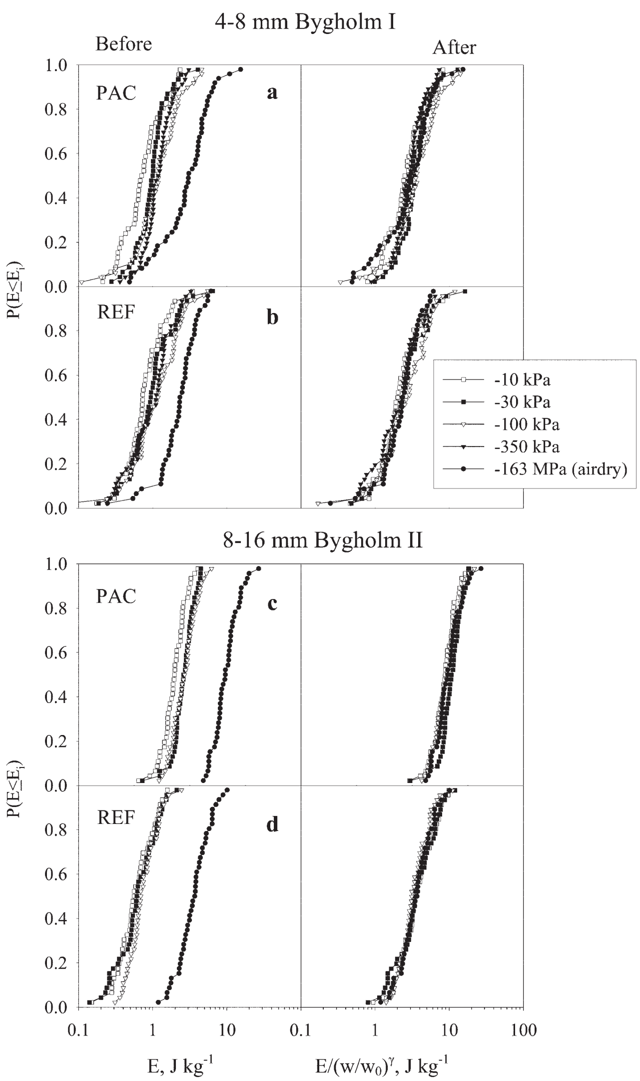

Fig. 3. Cumulative density functions for the cumulative probability density function $P(E \leq E_i)$ vs. specific rupture energy E before and after normalization of E to the air-dry state [$E(w/w_0)$] for (a) and (b) 4- to 8-mm Bygholm I aggregates, and (c) and (d) 8- to 16-mm Bygholm II aggregates. PAC = compacted treatment, and REF = reference treatment.

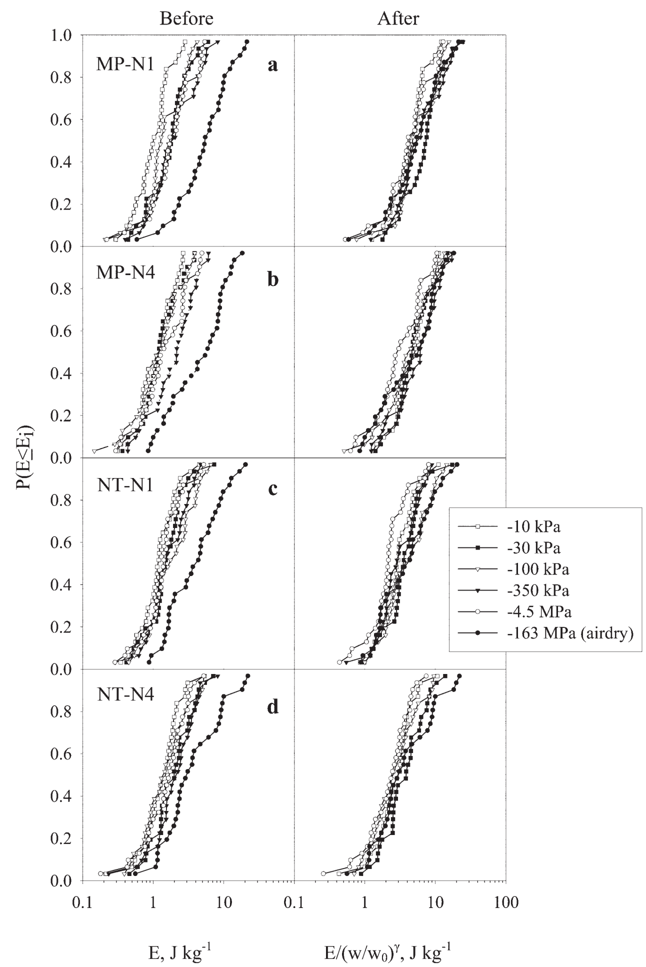

Fig. 4. Cumulative density functions for the cumulative probability density function $P(E \leq E_i)$ vs. specific rupture energy E before and after normalization of E to the air-dry state [$E(w/w_0)$] for Maury Block 2 data: (a) moldboard plowing and 0 kg N ha^{-1} yr^{-1} treatment (MP-N1), (b) moldboard plowing and 336 kg N ha^{-1} yr^{-1} treatment (MP-N4), (c) no-tillage and 0 kg N ha^{-1} yr^{-1} treatment (NT-N1), and (d) no-tillage and 336 kg N ha^{-1} yr^{-1} treatment (NT-N4).

Table 7. Regression coefficients: matric potential (Ψ), relative water contents (w/w_0), characteristic rupture energy (α), spread of aggregate strengths (β), rupture energy–aggregate size scaling factor (D), and R^2 values for the Eq. [4] Weibull model—Bygholm soils.

Soil/experiment	Treatment†	Ψ	w/w_0‡	α	β	D	R^2
				J kg^{-1}			
		−166 MPa	1.0	2.50 (0.05)§	1.64 (0.05)	1.27 (0.05)	0.94***
		−350 kPa	6.0	0.90 (0.02)	1.81 (0.05)	1.54 (0.05)	0.95***
Bygholm I	PAC	−100 kPa	8.9	0.98 (0.02)	1.71 (0.05)	1.51 (0.05)	0.95***
		−30 kPa	11.2	0.62 (0.01)	2.30 (0.06)	2.11 (0.06)	0.97***
		−10 kPa	12.2	0.45 (0.01)	1.71 (0.05)	1.96 (0.06)	0.97***
		−166 MPa	1.0	2.45 (0.07)	1.49 (0.06)	1.09 (0.05)	0.90***
		−350 kPa	6.1	0.88 (0.02)	1.51 (0.05)	1.37 (0.05)	0.93***
Bygholm I	REF	−100 kPa	9.1	1.08 (0.02)	1.49 (0.05)	1.21 (0.05)	0.94***
		−30 kPa	12.0	0.70 (0.01)	1.78 (0.05)	1.63 (0.05)	0.96***
		−10 kPa	13.3	0.42 (0.01)	1.80 (0.04)	2.20 (0.06)	0.97***
		−166 MPa	1	3.31 (0.04)	1.90 (0.04)	1.34 (0.04)	0.97***
Bygholm II	PAC	−100 kPa	9.4	0.64 (0.02)	1.75 (0.07)	0.99 (0.05)	0.89***
		−30 kPa	12.4	0.64 (0.01)	2.21 (0.04)	1.35 (0.03)	0.98***
		−10 kPa	14.2	0.48 (0.00)	2.05 (0.04)	0.96 (0.03)	0.98***
		−166 MPa	1	2.89 (0.05)	1.83 (0.05)	1.22 (0.05)	0.94***
Bygholm II	REF	−100 kPa	9.2	0.49 (0.01)	1.97 (0.06)	1.56 (0.05)	0.94***
		−30 kPa	12.5	0.48 (0.01)	1.61 (0.05)	1.27 (0.05)	0.94***
		−10 kPa	14.5	0.39 (0.01)	1.73 (0.05)	1.31 (0.05)	0.96***

*** Significant at the $P < 0.001$ level.

† PAC, compacted treatment; REF, reference treatment.

‡ Averaged across size fractions.

§ Approximate standard errors in parentheses.

sample. This means that a few dominant pores, rather than the overall pore structure, affect tensile failure. The stress concentration increases with increased length and narrowness of the cracks tips. In other words, tensile failure is expected to initiate in a long, narrow crack perpendicular to the direction of the loading (Hallett et al., 1995). It is obvious that the length of the longest pore or crack is expected to increase with aggregate length, and this may explain the low values of D obtained in this study. Tensile failure also depends on aggregate volume, however, because the larger the volume, the more pores and cracks can be accommodated.

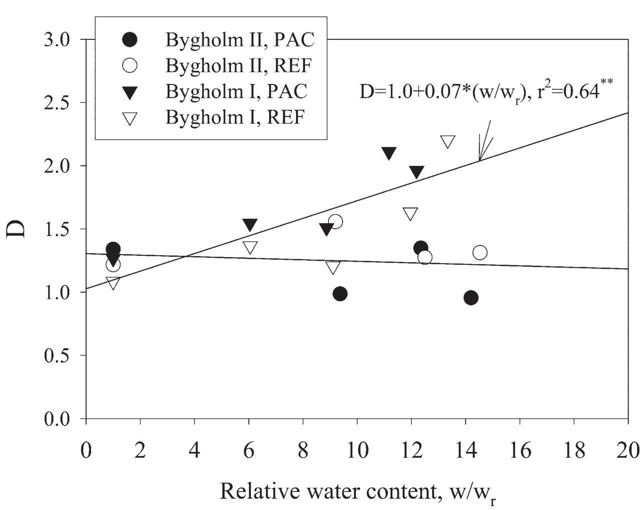

Fig. 5. Relationship between the rupture energy–aggregate size scaling factor (D) and the relative water content (w/w_0) for the Bygholm soils. The lines are best-fit linear regressions; the relationship for the Bygholm II soil was not significant. PAC = compacted, and REF = reference treatment.

Water content displayed different effects on D depending on the Bygholm soil. For the Bygholm I soil, D increased with increasing water content, whereas no significant correlation was found for the Bygholm II soil (Table 7, Fig. 5). An increase in D with increasing water content indicates an increased effect of pore area and volume, compared with length, in the failure process. For the Bygholm II soil, the D values were constantly low (i.e., $D < 1.5$) at all water contents. This behavior suggests that tensile rather than shear failure continued to be the dominant mode of failure with increasing water content in this soil. Based on our results, it is not clear if a correction for the relationship between water content and D should be included in a Weibull model that also takes aggregate size into account. Further research is needed to clarify the full extent of possible interactions between pore space geometry and water content in brittle fracture of soil aggregates.

CONCLUSIONS

In this study, water content was shown to strongly affect the characteristic rupture energy (Weibull α parameter). This relationship could be modeled by a power law function. In contrast, water content had little or no effect on the spread of aggregate strengths as parameterized by the Weibull β parameter. A significant positive relationship was found only in the Maury soil and not for all cases. We conclude that water content for the tested sandy loam and silt loam soils could be satisfactorily taken into account in the Weibull model for aggregate strength by including only the relationship between α and water content. For any given size fraction, the resulting three-parameter Weibull model explained a large part of the variation in all cases in this study. Water content displayed a positive (Bygholm I soil) or no effect (Bygholm II soil) on the rupture energy–aggregate size scaling factor, D. Potentially, only four Weibull parameters (α_0, β, γ, and D) will be needed to model rupture energy for soil aggregates across a range of water contents and size fractions. Further investigations are needed, however, to more fully test this hypothesis.

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