Vadose Zone Journal

Estimating Effective Hydraulic Parameters of Unsaturated Layered Sediments Using a Cantor Bar Composite Medium Model

Guoping Tang, Edmund Perfect,* Elmer H. van den Berg, Melanie A. Mayes, and Jack C. Parker

The estimation of effective hydraulic parameters of variably saturated layered sediments has been extensively studied using deterministic, stochastic, and combined modeling approaches. Heterogeneity and scale dependence remain as major obstacles, however, for the prediction of water flow and contaminant transport at many U.S. Department of Energy sites. We used a physically based Cantor bar model to describe scale dependence of the fractions of coarse and fine materials in layered sediments. The Cantor bar is determined by three fractal parameters: the subdivision factor, *b*, the fractal dimension, *D*, and the iteration level, *i*, which can be estimated from observation (e.g., borehole logs). Because *b* and *D* are scale invariant, the model can be used to predict layering at scales other than the observation scale. Together with a composite medium approximation (COMA), the Cantor bar model can be used to predict effective hydraulic properties as a function of scale. Numerical simulation results showed that COMA works well for steady-state unsaturated flow through a stratigraphic sequence comprised of thin layers of fine material interbedded within a coarse material. Further work is necessary to validate the model predictions by performing measurements at different scales, and to assess the applicability of this approach for transient flow.

Abbreviations: COMA, composite medium approximation; pb, parallel to bedding; xb, across bedding.

To address the environmental problem of leaking radioactive waste storage tanks at the U.S. Department of Energy's Hanford site in the state of Washington, tremendous efforts have been devoted to characterizing the layered sediments, estimating the effective hydraulic properties, and simulating the migration of moisture, heat, and radionuclides in the subsurface (e.g., Pruess and Yabusaki, 2002; Khaleel, 2004). Heterogeneity and scale dependence remain major obstacles, however, for the prediction of water flow and contaminant transport in the vadose zone at USDOE sites. The objective of this work was to introduce a physically based fractal model to characterize geological heterogeneity and to estimate effective hydraulic properties for layered sediments at scales of interest.

Sufficient deterministic characterization of heterogeneity is not feasible and statistical investigations are limited by the scarcity of data. Traditional approaches focus on the observation scale and underutilize the results of geologic investigations. This has stimulated the development of heterogeneity models based on geology (e.g., geologic structure, Webb and Anderson, 1996; fluvial bounding surfaces, Davis et al., 1997; and facies, Allen-King et al., 1998). The present work uses a fractal model to represent geological heterogeneity.

Observing a cliff face or outcrop from a distance, a number of units might be recognized. Getting closer, each individual unit can be further subdivided into smaller units. Skipping a few intermediate steps and zooming in on a microscopic scale, laminae may be identified, each having unique characteristic properties and thicknesses. According to this scenario, the thickness distribution of layered sediments should exhibit fractal characteristics. Mandelbrot and Wallis (1969) were the first to analyze bed thickness series in a stratigraphic sequence using fractals. Since then many researchers have applied fractal techniques to both parameterize (e.g., Hewett, 1986; Tubman and Crane, 1995; Richards et al., 2000; Richards, 2002; Bailey and Smith, 2005) and simulate (e.g., Plotnick, 1986, 1988; Plotnick and Prestegaard, 1995; Schlager, 2004) the frequency and magnitude of layers in stratigraphic sequences.

Fractals are geometric models comprised of irregular or fragmented elements that repeat themselves across a wide range of scales. Since fractal parameters are scale invariant, they are a natural choice for inclusion in physically based models designed to predict properties or processes at one scale based on information collected at another scale. A fractal model frequently used for simulating the layer thickness distribution in a stratigraphic sequence is the Cantor bar model (Plotnick and Prestegaard,

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1995; Turcotte, 1997). With fractal parameters estimated at the observation scale, the Cantor bar model can be used to predict the heterogeneity and thickness distribution of layers at different scales of interest.

Both deterministic and stochastic methods have been used to upscale unsaturated hydraulic parameters for heterogeneous porous media. Khaleel (2004) and Khaleel et al. (2003) used RETC (van Genuchten et al., 1991) to fit composite parameters to experimental data and estimated the anisotropy using a stochastic approach proposed by Polmann (1990). Pruess and Yabusaki (2002) used a COMA (Pruess, 2004) similar to that developed for the saturated flow case. This direct averaging approach has been discussed by many researchers (e.g., Mualem, 1984; Yeh et al., 1985; Yeh and Harvey, 1990; Khaleel et al., 2002). However, it is difficult to assess its accuracy and range of applicability (Pruess, 2004). Khaleel et al. (2002) suggested that it exaggerates the variability of conductivity and yields unrealistic estimations by ignoring the gradient variance. While experimental results by Stephens and Heermann (1988), Ellsworth et al. (1991), and Glass et al. (2005) suggested moderate anisotropy, both deterministic and stochastic approaches predict a much larger anisotropy, particularly for dry conditions (e.g., Mualem, 1984; Yeh et al., 1985; Green and Freyberg, 1995; Pruess, 2004). The anisotropy is directly related to the lateral spreading of contaminants in layered sediments. Therefore, it is necessary to assess the validity of these upscaling techniques.

Our goal was to improve effective hydraulic parameter estimation for layered sediments at different scales. First, the Cantor bar composite medium model is described. We conducted numerical simulations to assess the COMA approach. Some limitations and further work are discussed.

Theory

A Cantor Bar Model for Layered Sediments

The Cantor bar is a simple monofractal. The standard Cantor bar is presented in Fig. 1. It is generated from a bar (initiator) of unit length by repeatedly removing the middle $1/b$ of each existing bar, where b is the subdivision factor. In general, n parts of $1/b$ can be removed from random locations of each existing bar. By varying b, n, and the removal locations, the Cantor bar model can be used to describe a wide range of layered systems.

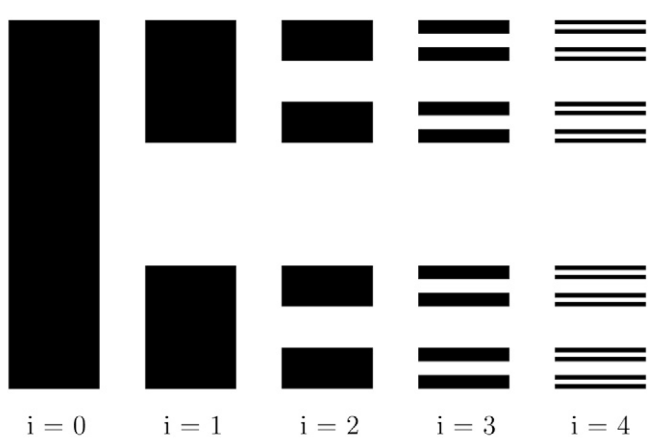

Fig. 1. Standard Cantor bar (subdivision factor b = 3 and fractal dimension D = 0.6309).

Two representative materials need to be selected for the Cantor bar model to describe the sediment sequence. In reality, though, there may be more than two materials present, as discussed below. Nevertheless, many types of sediment can be approximated by a sequence of fine and coarse materials. For a sedimentary sequence with total thickness of L_T, when the observation is far away and no layers can be seen, $i = 0$. At a specific observation scale, b can be approximated by $b = L_T/L_{max}$, with L_{max} as the thickness of the thickest layer. The thickness of the thinnest discernible layer (L_{min}) defines i with $b^{-i} = L_{min}/L_T$ at the observation scale. Therefore, i represents the scale in the Cantor bar, and its value may be any positive real number.

The parameters n and b determine the fractal dimension, $D = \ln(b - n)/\ln(b)$, which is independent of scale. In fractal geometry, the fractal dimension gives an indication of how efficiently an irregular or fragmented system fills the available Euclidean space as one zooms down to finer and finer scales. In this application, it controls the number of elements, N_1, of the starting material at iteration i, i.e.,

$$N_1 = \left(\frac{1}{b^i}\right)^{-D} \qquad [1]$$

Bailey and Smith (2005) showed that layer thickness and frequency of occurrence in many stratigraphic sequences can be described by Eq. [1]. The fractal dimension, D, is the most widely estimated parameter in any application of fractals. A number of techniques are available for estimating D from borehole data, including spectral analysis, rescaled range analysis, and variogram analysis (Turcotte, 1997; Plotnick and Prestegaard, 1995). Based on spectral analysis, Tubman and Crane (1995) and Richards (2002) reported fractal dimensions of about 0.17 for vertically oriented logs in a braided-stream sandstone deposit and 0.3 for a glaciodeltaic cross-bedded sand deposit, respectively.

Because b and D are scale invariant, the Cantor bar model can be used to predict the thickness distribution of layers at scales other than the observation scale. For example, considering the layered system with L_T = 10 m described by the standard Cantor bar presented in Fig. 1, the case i = 4 corresponds to a length scale of $10/3^4$ = 0.12 m, which is close to the observation scale of borehole logs. Then the case of i = 2 can be seen as a prediction of the layered system at the length scale of $10/3^2$ = 1.1 m, which can be the grid size scale required for numerical simulations. While a smaller i is used for upscaling, a larger i can be used for downscaling; however, there will be a physical upper limit for i determined by the thinnest lamina. With the Cantor bar model describing the thickness distribution of layered sediments, the COMA can be used for hydraulic parameter estimation.

The Composite Medium Approximation

As described by Pruess (2004), composite unsaturated hydraulic properties for a perfectly layered sediment with M layers can be approximated by

$$\theta_{comp}(h) = \sum_{j=1}^{M} f_j \theta_j(h) \qquad [2]$$

$$k_{comp,pb}(h) = \sum_{j=1}^{M} f_j k_j(h) \qquad [3]$$

and

$$k_{\mathrm{comp,xb}}(h)=\left[\sum_{j=1}^{M}\frac{f_j}{k_j(h)}\right]^{-1} \quad [4]$$

where $\theta_{\mathrm{comp}}(h)$, $k_{\mathrm{comp,pb}}(h)$, and $k_{\mathrm{comp,xb}}(h)$ are the composite volumetric moisture content [$L^3\ L^{-3}$] and hydraulic conductivity [$L\ T^{-1}$] parallel to bedding (pb) and across bedding (xb), respectively; h is the pressure head [L] (gauge pressure head as measured with a manometer); and f_j, θ_j, and k_j are the fraction, moisture content, and hydraulic conductivity of layer j. The fraction of the starting material, f_1, in the Cantor bar model is determined by

$$f_1=\frac{N_1}{b^i}=\left(\frac{1}{b^i}\right)^{1-D} \quad [5]$$

A bimodal stratigraphic sequence can best be modeled by thin layers of fine sediments separated by coarse sediments of variable thickness. Thus, it is assumed that f_1 in the Cantor bar model is the fraction of the fine material (i.e., the black phase in Fig. 1). From Eq. [5], $f_1 = 1$ at $i = 0$. With increasing i or b or decreasing D, f_1 decreases.

Effective Hydraulic Parameters

Steady-state vertical unsaturated water flow is described by the Buckingham–Darcy equation (Narasimhan, 2004) as

$$q=-k\left(\frac{\mathrm{d}h}{\mathrm{d}z}+1\right) \quad [6]$$

where q is the Darcy velocity [$L\ T^{-1}$] and z is the elevation. For a domain of [0, L] with constant $h(0) = h_0$ and $h(L) = h_L$, the effective (block) hydraulic parameters are defined as

$$q=-k_{\mathrm{eff}}\left(\frac{h_L-h_0}{L}+1\right) \quad [7]$$

$$\theta_{\mathrm{eff}}=\int_0^L\theta\frac{\mathrm{d}z}{L} \quad [8]$$

where θ is the moisture content distribution in the domain, θ_{eff} is the effective moisture content, which is an arithmetic mean of θ, and k_{eff} is the effective k, which is assumed to be isotropic at small scales. Rearranging Eq. [6] results in

$$-\frac{q}{k}-1=\frac{\mathrm{d}h}{\mathrm{d}z} \quad [9]$$

Integrating both sides,

$$-\int_0^L\left(\frac{q}{k}+1\right)\mathrm{d}z=h_L-h_0 \quad [10]$$

under steady-state conditions where q is constant, and replacing $h_L - h_0$ in Eq. [7] with Eq. [10] results in

$$k_{\mathrm{eff}}=\left(\int_0^L\frac{1}{k}\frac{\mathrm{d}z}{L}\right)^{-1} \quad [11]$$

Therefore, k_{eff} is the harmonic mean of k at points in the domain from [0, L]; $k_{\mathrm{eff}} = k$ for sufficiently small L, or uniform h, or state-independent k. For unsaturated flow, these conditions are generally not valid. Instead, k_{eff} is dependent on L, h_0, h_L, and the distribution of h. Therefore, k_{eff} is both scale and state dependent. Even though the equations here are for vertical flow, the same results can be reached for horizontal flow or any inclination by replacing the gravity term 1 in Eq. [6] by $\cos\beta$, with β as the inclination.

For a perfectly layered sediment that consists of M layers with thickness ΔL_j for layer j, then for xb flow, Eq. [11] becomes

$$k_{\mathrm{eff,xb}}=\left(\sum_{j=1}^{M}\int_{L_{j-1}}^{L_j}\frac{1}{k}\frac{\mathrm{d}z}{\Delta L_j}\right)^{-1} \quad [12]$$

Denoting $k_{\mathrm{eff},j}=\left(\int_{L_{j-1}}^{L_j}\frac{1}{k}\frac{\mathrm{d}z}{\Delta L_j}\right)^{-1}$ and $f_j=\frac{\Delta L_j}{L}$, then

$$k_{\mathrm{eff,xb}}=\left(\sum_{j=1}^{M}\frac{f_j}{k_{\mathrm{eff},j}}\right)^{-1} \quad [13]$$

Therefore, $k_{\mathrm{eff,xb}}$ is the harmonic mean of k_{eff} in individual layers. For pb flow and with the same boundary pressure heads in each layer, the pressure head distributions in the neighboring layers in general are not identical; therefore, there will be flow across the interfaces and the flow is actually two dimensional. Computation of the pb effective hydraulic conductivity is complicated. If the across-interface flow is ignored, however, a simple one-dimensional relation can be reached as

$$k_{\mathrm{eff,pb}}=\sum_{j=1}^{M}k_{\mathrm{eff},j}f_j \quad [14]$$

with $k_{\mathrm{eff},j}$ defined as in Eq. [11] and L now the distance in the horizontal direction.

Comparing Eq. [3] with Eq. [14] and Eq. [4] with Eq. [13] indicates that $k_j = k_{\mathrm{eff},j}$. This may be valid in the case of a very small representative volume or small variance in pressure head, hydraulic conductivity, gradient, or flow rate. Namely, the COMA ignores the pressure variance in individual layers. Note also that the composite hydraulic parameters are scale independent. In addition, only the fraction of the different sediment types has an impact on the composite parameters. The actual distribution does not matter. To further assess how well the COMA approximates the layered sediments in terms of unsaturated hydraulics, numerical simulations were conducted using a Cantor bar model.

Numerical Simulations

Hydraulic Parameters

To illustrate the concept of the Cantor bar composite medium model, the unsaturated hydraulic parameters for two representative Hanford sediments from Pruess (2004) were used. The parameters are listed in Table 1, with the van Genuchten model (van Genuchten et al., 1991) defined as

$$S_{\mathrm{e}}=\frac{\theta-\theta_{\mathrm{r}}}{\theta_{\mathrm{s}}-\theta_{\mathrm{r}}}=\left[1+\left(\alpha|h|\right)^n\right]^{-m}$$

$$k_{\mathrm{r}}=\frac{k}{k_{\mathrm{s}}}=\frac{\left\{1-\left(\alpha|h|\right)^{n-1}\left[1+\left(\alpha|h|\right)^n\right]^{-m}\right\}^2}{\left[1+\left(\alpha|h|\right)^n\right]^{m/2}} \quad [15]$$

where θ_{r} and θ_{s} are residual and saturated moisture content, respectively; k_{r} and k_{s} are relative and saturated hydraulic conductivity,

TABLE 1. The van Genuchten parameters of saturated hydraulic conductivity (K_s), residual and saturated volumetric water content (θ_r and θ_s, respectively), and shape factors α and n for representative fine and coarse Hanford sediments (Pruess, 2004).

Sediment	K_s	θ_r	θ_s	α	n
	cm s^{-1}			cm^{-1}	
Fine	3.70×10^{-4}	0.0300	0.3586	0.0092	1.8848
Coarse	3.53×10^{-2}	0.0367	0.3309	0.0395	2.6308

respectively; and α [L^{-1}], n, and $m = 1 - 1/n$ are van Genuchten parameters.

For the standard Cantor bar model (Fig. 1) starting with the fine material, the composite parameters computed using COMA are presented in Fig. 2 with $i = 0$ for the fine material, $i = \infty$ for the coarse material, and i = 1, 3, and 5 for the composite cases. As i increases, the $k_{comp,pb}$ curve (Fig. 2a) moves from the curve for the fine material to the curve for the coarse material, which is due to the coarse material fraction increasing as i increases. The $k_{comp,pb}$ for the intermediate iterations is bounded by the values of the two representative materials. The reason is that the arithmetic mean is used for both $k_{comp,pb}$ and θ_{comp}. The $k_{comp,xb}$ curves (Fig. 2b) for intermediate iterations are not bounded by the hydraulic conductivity curves of the two materials. The reason is that $k_{comp,xb}$ is computed using the harmonic mean while θ_{comp} is calculated using the arithmetic mean. If plotted against h, they will be bounded as shown in Pruess (2004). The anisotropy, $k_{comp,pb}/k_{comp,xb}$, is shown as a function of tension in Fig. 2c. The predicted curves are similar to those presented by many researchers (e.g., Mualem, 1984; Pruess, 2004). It seems that the anisotropy is not very sensitive to the iteration or observation scale. This suggests that the presence of a sediment layer with different hydraulic parameters can induce significant anisotropy, even if the fraction is relatively small.

In practice, the fractal and hydraulic parameters are only available at the observation scale; for example, a 10-cm observation scale is associated with $i = 3$. The hydraulic parameters at $i = 1$ are then a prediction of the upscaled parameters and may be used for numerical simulation with a grid length of 1 m. The $i = 5$ case can be seen as an example of downscaling. Note that the composite hydraulic parameters are scale dependent even though COMA is scale independent. Therefore, theoretically, the Cantor bar composite medium model can be used to predict hydraulic parameters at different scales; however, validation by measurements at different scales is needed to validate the predictions. Also, numerical experiments are required to assess the COMA approximation.

Numerical Solution

To assess COMA, full-resolution simulations were conducted to compare the effective parameters computed with composite (upscaled using COMA) and actual layered parameters. The standard Cantor bar models the thickness distribution of layers. The iteration starts with the fine sediment ($i = 0$). To investigate the influence of scale, two lengths L = 10 and 100 cm are considered, with L = 10 cm close to the observation scale (bore logging) and L = 100 cm close to a desirable grid size for numerical simulation. The domain is discretized with a uniform grid size of $L/243$ (corresponding to $b^i = 3^5$ for the standard Cantor bar). The boundary conditions are: no flux on either the left or right sides, constant pressure h_0 at the bottom, and constant flow rate q at the top. Gravity is ignored.

Because only one-dimensional, steady-state flow is considered, Eq. [10] is integrated numerically to obtain the solution instead of solving the transient Richards equation (e.g., Simunek et al., 2005). The integration starts from the bottom and marches upward from grid node to grid node. The Picard iteration method (e.g., Reddy, 2004) is used for linearization with a tolerance of 10^{-3} cm. An adaptive Lobatto quadrature (Gander and Gautschi, 2000) is used to compute $\int_{z_{j-1}}^{z_j} 1/k\,dz$ for each grid with a tolerance of 10^{-3} times the previous integration value. For dry or

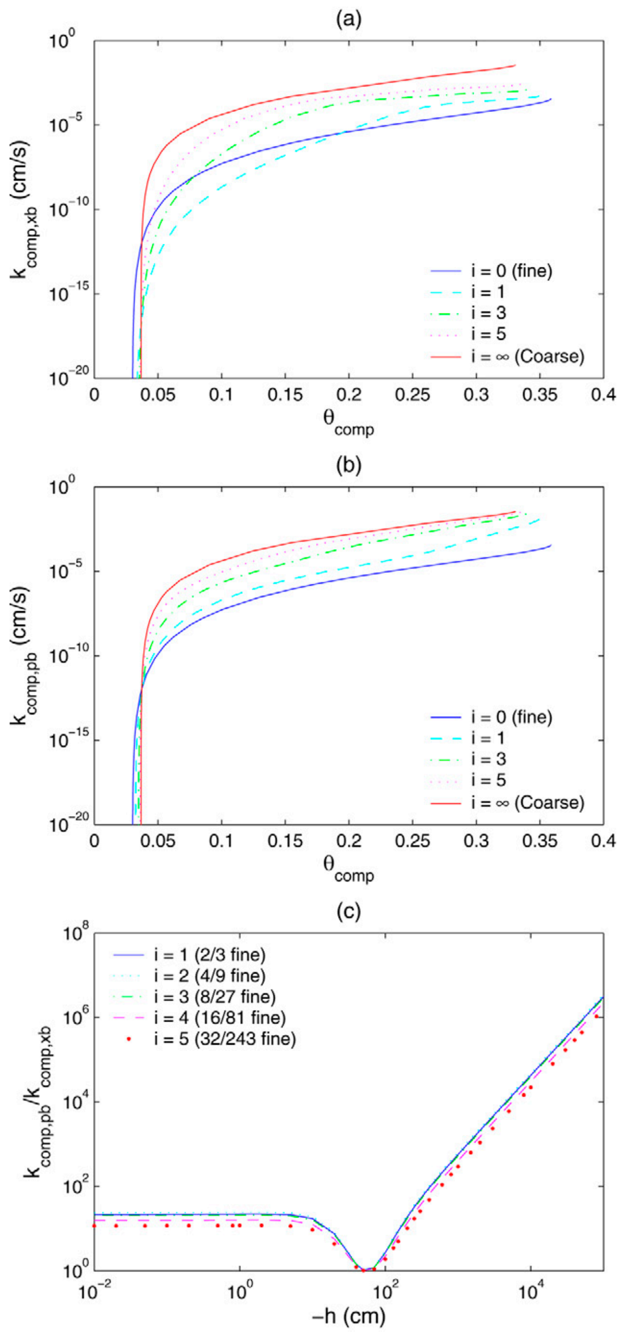

FIG. 2. Composite hydraulic parameters for the standard Cantor bar model: k = hydraulic conductivity; θ = volumetric water content; comp = composite; pb = parallel to bedding; xb = across bedding; h = pressure head; i = iteration; fine = fine materal; coarse = coarse material.

high-gradient cases, k can be very close to zero, which can cause convergence problems. The h distribution is assumed to be linear in each grid because the h profiles are continuous and smooth. With the solution, Eq. [7] and [8] are used to compute the effective parameters. The gradient is $(h_0 - h_L)/L$ with h_L as the pressure at the top. The results were partially confirmed by using HYDRUS-1D (Simunek et al., 2005). For the high-gradient and large length scale cases, convergence difficulties also arose with HYDRUS-1D.

Results

The effective parameters computed at various gradients are compared with the composite parameters in Fig. 3 for the case of $i = 3$ and $L = 10$ cm. For a unit gradient (gravity flow, $\Delta h = 10$ cm), the layered effective (computed using layered hydraulic parameters), composite effective (calculated using composite hydraulic parameters), and COMA (same as shown in Fig. 2) results are almost the same. As the gradient increases, the effective parameters deviate from the COMA predictions. The differences are small for dry conditions and increase with increasing moisture content. Therefore, the gradient has a significant influence on the effective (block) hydraulic parameters. The layered effective parameters, however, are close to the composite effective parameters, with the hydraulic conductivity of the layered case being slightly greater than that of the composite case for moist conditions or high gradients. With the same constant pressure at both boundaries, the average moisture content for the layered case is smaller than that of the composite case. Also, the difference is very small for dry conditions and increases with moisture content. The anisotropy curves for both the effective and composite cases are close for different gradients (with slightly smaller anisotropy for high gradients and moist conditions) despite the fact that the effective hydraulic conductivity is generally less than that for the composite parameters at high gradients and high moisture content conditions.

The h, θ, and k profiles for the xb flow case with $h_0 = -100$ cm are compared in Fig. 4. As described by Narasimhan (2004), the gradient is gentler at the inlet and steeper at the outlet. The h profile at unit gradient ($\Delta h = 10$ cm) is the same for both the composite and the layered case. For a high gradient (or Δh), the differences are mainly at the very top region. Therefore, only the top half of the profile is shown. This results in the same trend for the θ and k distributions. The moisture content for the composite case is close to the arithmetic mean of that of the layered case, while the hydraulic conductivity of the composite case is close to the harmonic mean of that of the layered case. The variance in the pressure or hydraulic conductivity at high gradients causes the difference between the effective and composite hydraulic conductivities.

With the same boundary conditions (h_0 and h_L), the solution (h distribution) for the $L = 100$ cm case is very close to that of the $L = 10$ cm case. The effective hydraulic parameters are similar to those presented in Fig. 3; the profiles are similar to those shown in Fig. 4, but the vertical axis ranges from 0 to 100 cm instead of 10 cm. Therefore, the results are not presented. The gradient, though, of the $L = 10$ cm case is 10 times that of the $L = 100$ cm case. Besides that, the influence of block length is minor. In summary, the numerical simulation results for the composite cases are very close to that for the layered case, confirming that the COMA works well at least for steady-state conditions regardless of the block length and gradient.

Discussion and Conclusions

A Cantor bar model was used to characterize the heterogeneity of layered sediments based on the fractal parameters D, b, and i. The fractal dimension D determines the relative propor-

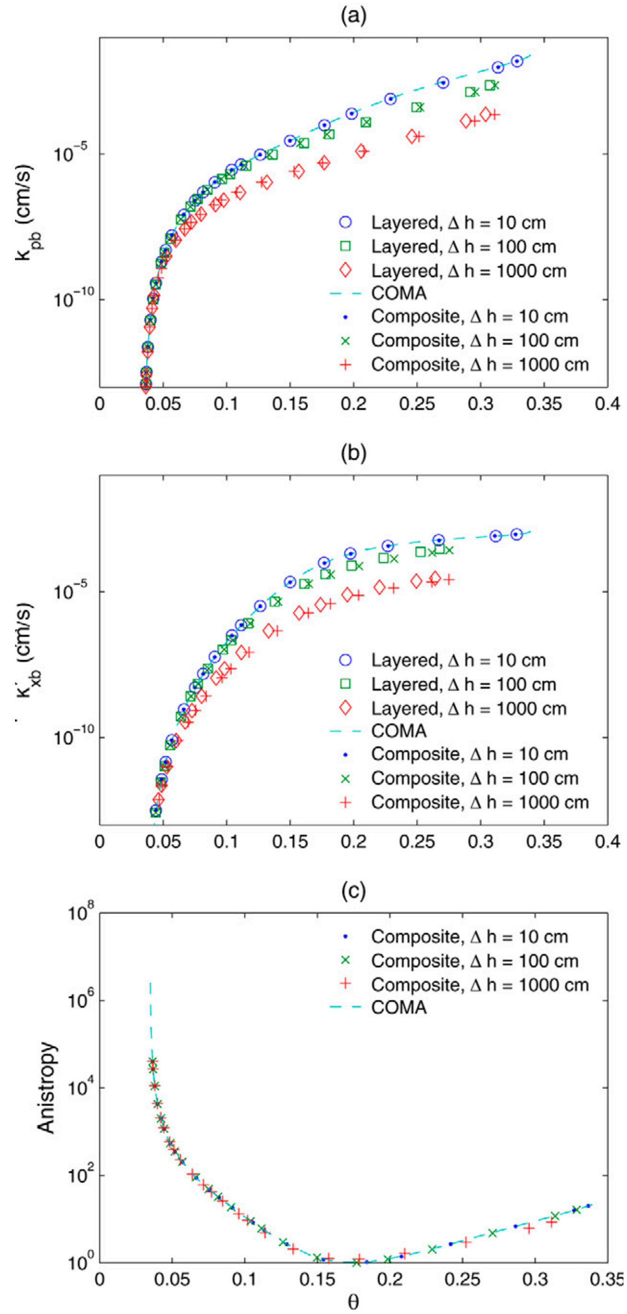

FIG. 3. Comparison between the composite and effective parameters x for a standard Cantor bar with iteration $i = 3$; k_{pb} = hydraulic conductivity for flow parallel to bedding; k_{xb} = hydraulic conductivity for flow across bedding; θ = volumetric water content; Δh refers to the pressure head difference; layered refers to effective parameters calculated using full-resolution solution with actual hydraulic parameters; composite refers to effective parameters calculated using the solution with the composite parameters from composite medium approximation (COMA).

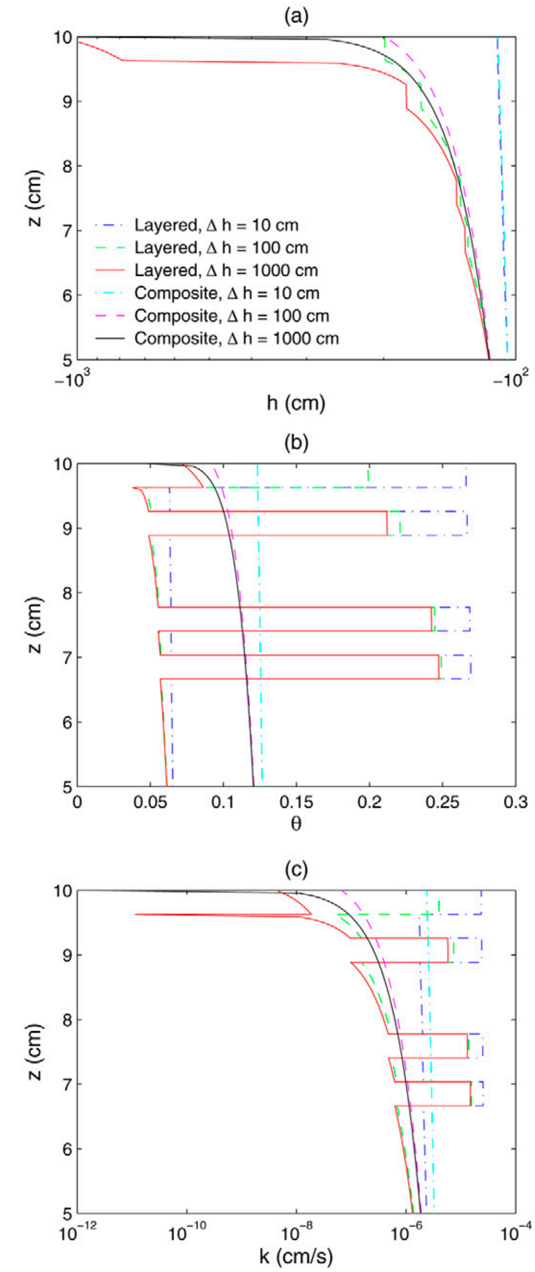

FIG. 4. Pressure head, h, hydraulic conductivity, k, and volumetric water content, θ, profiles for cross-bedding flow (standard Cantor bar with iteration i = 3 and pressure head at the bottom h_0 = −100 cm); Δh refers to the pressure head difference between the top and bottom boundaries; z = depth; layered refers to effective parameters calculated using full-resolution solution with actual hydraulic parameters; composite refers to effective parameters calculated using the solution with the composite parameters from composite medium approximation (COMA).

tions of fine and coarse sediments at different scales. The b parameter represents the ratio of the total length of the stratigraphic sequence to the thickness of the thickest layer present. The i parameter is related to the scale. With the fractal and hydraulic parameters estimated at an observation scale, the Cantor bar model can be used to predict composite hydraulic properties at other scales of interest using the COMA. Validation of such predictions will depend on the availability of measurements made at different scales. To use this approach for forward prediction at the USDOE's Hanford site, it will be necessary to estimate D from borehole data. Fractal analyses of stratigraphic sequences performed by Richards (2002) and Bailey and Smith (2005) provide examples of what is possible.

Although there are several approaches that can be used to simulate geologic sequences of layers (Tetzlaff and Harbaugh, 1989), the Cantor bar fractal model for layered heterogeneity appears to possess a number of advantages over geological process models or stochastic methods that are conditioned to the observed lithology and thickness distribution. For example, it allows prediction at any scale or vertical depth interval of the pattern of layering and the thickness size distribution in such a way that the statistical properties are consistent. Furthermore, since key sedimentary processes, e.g., sea level changes (Fluegeman and Snow, 1989; Hsui et al., 1993; Harrison, 2002), flood frequencies (Malamud and Turcotte, 2006), and sediment transport (Shang and Kamae, 2005) have been shown to exhibit fractal behavior, it has the potential to link spatial and temporal heterogeneities. This could help explain the time sequence of depositional events responsible for a particular stratigraphic sequence. An additional advantage is that the limited number of fractal parameters need not necessarily be determined for that particular scale. These attributes make the Cantor bar an attractive tool for upscaling physical properties that are dependent on layering.

The whole sequence of the Hanford sediments may not be fully characterized by a single Cantor bar model with just two materials and a single fractal dimension. In reality, the sequence might be divided into several subunits, each with its own pair of contrasting materials and fractal parameters. The COMA model could then be implemented to compute composite hydraulic parameters for the entire sequence of sediments or for individual subunits, depending on the scale of interest.

A major limitation with the present model is that it is restricted to layers of just two different materials. This is because it is a monofractal model. In reality, many different materials can occur in a natural stratigraphic sequence. A multifractal model (Plotnick and Prestegaard, 1995; Turcotte, 1997) could be used to accommodate multiple material types in future research. Recently, Perfect et al. (2006) derived an analytical expression for the effective saturated hydraulic conductivity of a random multifractal Sierpinski carpet. A similar approach could be used to predict anisotropic composite properties for a multifractal Cantor bar representing a saturated stratigraphic sequence. For unsaturated conditions, however, multiple parameters (e.g., k_s, θ_r, θ_s, α, m, and n, in the case of the van Genuchten model) are needed to fully characterize the hydraulic properties of each material. In this case it is not immediately clear which hydraulic parameter or parameters should be associated with the mass fractions in the multifractal. Additional research is needed to resolve this dilemma.

The Cantor bar model provides a unique scale-invariant configuration for layered sediments to examine COMA. Numerical experiments were conducted to compare the effective hydraulic parameters with the composite parameters at two length scales (10 and 100 cm). The results show that the composite hydraulic parameters are close to the effective parameters when the gradient is small or the moisture content is low. The differences increase with increasing gradient, moisture content, and length scale. Effective parameters computed using the composite parameters are very close to those computed using actual layered parameters, however, which suggests that the COMA works well at least for steady-state flow. Additional numerical simulations for transient flow are necessary to further evaluate the proposed approach. Also, the model is limited to perfectly layered cases. Considerations of the presence of permeable cracks or irregular formation are essential for practical application of the Cantor composite medium model.

Acknowledgments

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